

KCS Mathematics Department



$\frac{35}{77} = 45\%$

4th Form Paper 1 (Non-Calculator)

June 2012

1½ hours

Instructions

- Write your name in the space below and circle the initials of your teacher
- Answer ALL Questions
- Show all necessary working
- Write your answers in the spaces provided
- Where appropriate give answers to three significant figures
- Calculators may NOT be used
- There are 77 marks in total

Name: Hawkins-Hodder.....

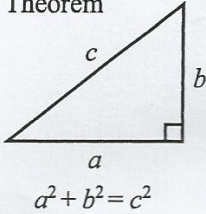
Teacher: HB SMB BJD KNH

TPH GDK GMCG SJN

AJP MPS TRS SUW

**IGCSE MATHEMATICS
FORMULAE SHEET – HIGHER TIER**

Pythagoras' Theorem

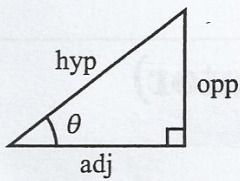
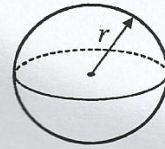
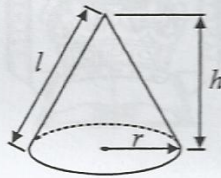


Volume of cone = $\frac{1}{3} \pi r^2 h$

Volume of sphere = $\frac{4}{3} \pi r^3$

Curved surface area of cone = $\pi r l$

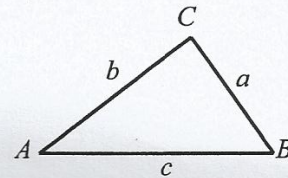
Surface area of sphere = $4 \pi r^2$



adj = hyp \times cos θ
opp = hyp \times sin θ
opp = adj \times tan θ

In any triangle ABC

or $\sin \theta = \frac{\text{opp}}{\text{hyp}}$



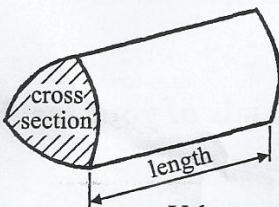
$\cos \theta = \frac{\text{adj}}{\text{hyp}}$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

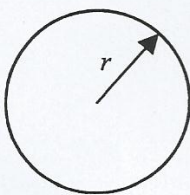
$\tan \theta = \frac{\text{opp}}{\text{adj}}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$



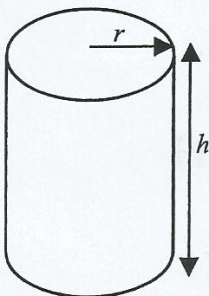
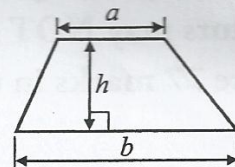
Volume of prism = area of cross section \times length



Circumference of circle = $2 \pi r$

Area of circle = πr^2

Area of a trapezium = $\frac{1}{2} (a + b) h$



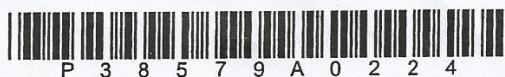
Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2 \pi r h$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



1. Solve the simultaneous equations

$6x - 5y = 13$
 $4x - 3y = 8$
 $12x - 10y = 26$
 $12x - 9y = 24$
 $-19y = 2$
 $-2 = 19y$
 $y = -\frac{2}{19}$
 $x = \frac{1}{19}$
 $x = 0.5$
 $18x - 15y = 39$
 $20x - 15y = 40$
 $2x = 1$
 $x = 0.5$
 ~~$18x - 15y = 39$~~
 ~~$20x - 15y = 40$~~
 ~~$-18x + 15y = -39$~~
 ~~$-20x + 15y = -40$~~
 ~~$-38x = -79$~~
 $3 - 5y = 13$
 $4x - 5y = 10$
 $y = 5.2 = 36.5$
 $x = 0.5$
 $y = -2$
 (Total 4 marks)

2. The table shows the first three terms of a sequence.

Term number	1	2	3	4	5
Term	2	5	10	17	26

The rule for this sequence is

Term = (Term number)² + 1

(a) Work out the next two terms of this sequence.

$\dots 17, 26 \dots$
 (2)

(b) One term of this sequence is 101. Find the term number of this term.

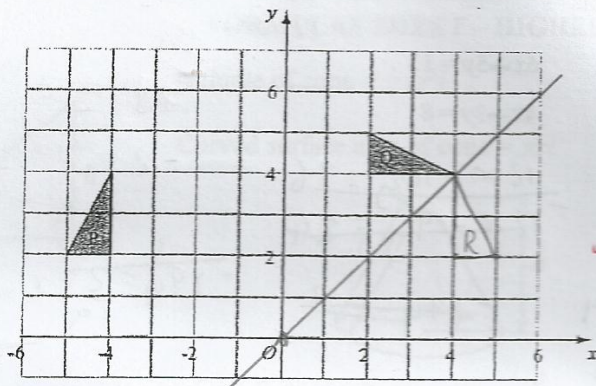
$101 - 1 = 100$
 $\sqrt{100} = 10$

$\dots 10 \dots$
 (2)

(Total 4 marks)

(8)

3.



(a) Describe fully the single transformation which maps triangle P onto triangle Q.

90° clockwise around the point $(0, 0)$

(3)

(b) Reflect triangle Q in the line

$y = x$

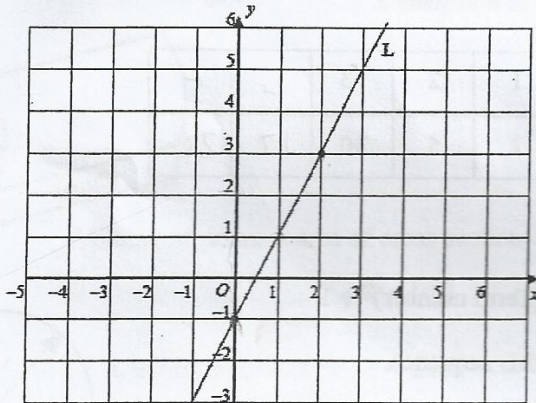
Label the new triangle R.

(4)

(2)

4.

The straight line, L, passes through the points $(0, -1)$ and $(2, 3)$.



(a) Work out the gradient of L.

$y = -1x + c$
 $y = mx + c$

$0, -1$ $2, 3$

$\frac{-2}{2} = -1$

$2 = -1 \times 3 + c$
 $+2 = -3 + c$
 $c = 5$

$y = -1x + 5$ (2)

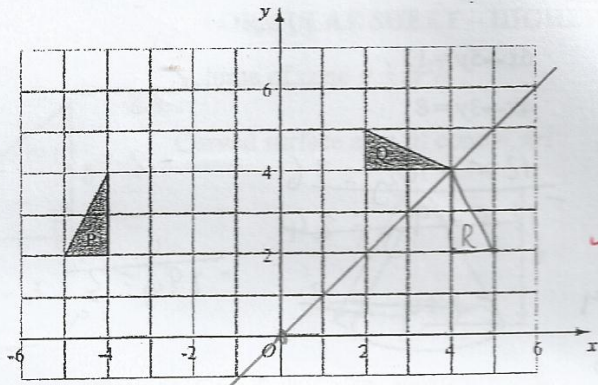
(b) Write down the equation of L.

$y = -1x + 5$ (2)

(c) Write down the equation of another line that is parallel to L.

$y = -1x + 6$ (1)

3.



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90° clockwise around the point (0, 0)

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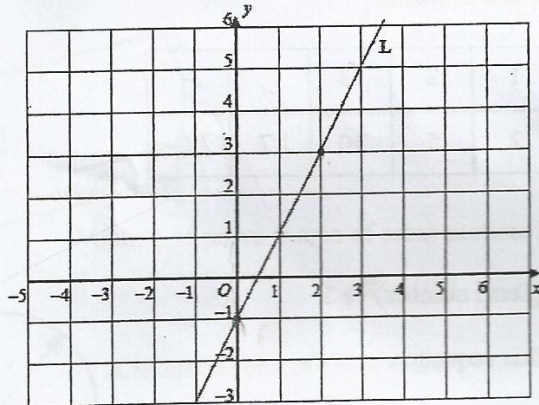
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0, -1 2, 3

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$y = -1x + 5$ (2)

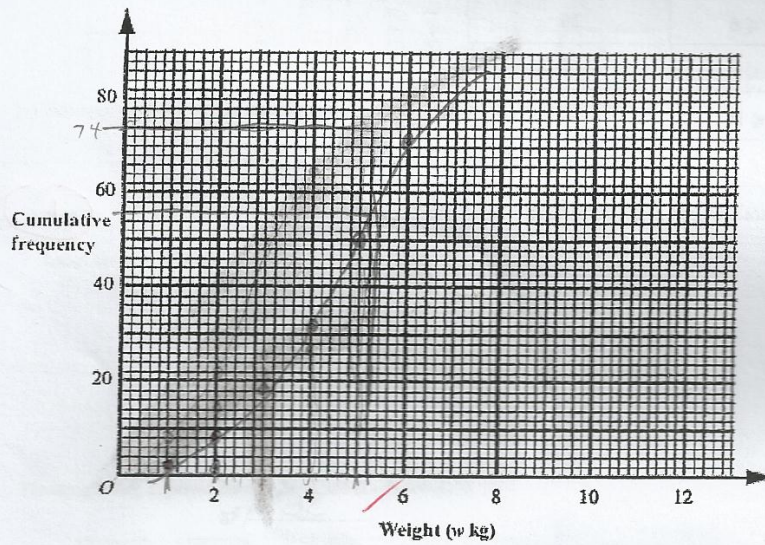
(b) Write down the equation of L.

$y = -1x + 5$ (2)

(c) Write down the equation of another line that is parallel to L.

$y = -1x + 6$ (1)

(c) On the grid, draw a cumulative frequency graph for your table.



(2)

(d) Use the graph to find an estimate for the number of parcels which weighed less than 5.2 kg.

34 56
(2)

(Total for Question 8 marks)

6. Six numbers have a mean of 5
Five of the numbers are

3 2 7 6 2 x

The other number is x .

Work out the value of x .

$$\underbrace{3 + 2 + 7 + 6 + 2}_{5} = 20$$

$$\frac{25}{5} - 20 = 5$$

$$3 + 2 + 7 + 6 + 2 + 5 = \frac{25}{5} = 5$$

$$x = 5$$

(Total for Question 2 marks)

7. (i) Solve the inequality $2x + 13 \geq 6$

$$2x \geq -7$$

$$x \geq -3.5$$

$$\underline{x \geq -3.5}$$

- (ii) n is a negative integer.

Write down all the values of n which satisfy $2n + 13 \geq 6$

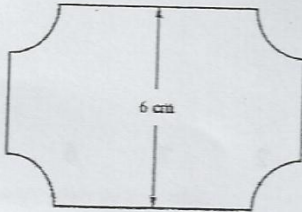
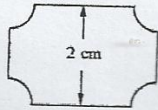
$$-6.5 + 13 \geq 6$$

$$\underline{-3, -2, -1}$$

(Total for Question 4 marks)

(4)

8. Here are two supermarket price tickets.



Diagrams NOT accurately drawn

The two supermarket price tickets are mathematically similar.

The area of the smaller ticket is 7 cm^2 .
Calculate the area of the larger ticket.

$$6 \div 2 = 3$$

$$\begin{aligned} \text{LSF } 3 \\ \text{ASF } 3^2 = 9 \\ \text{VSF} \end{aligned}$$

$$7 \times 9 = 63$$

$$\underline{63} \text{ cm}^2$$

✓ ②

(Total for Question 2 marks)

Solve the simultaneous equations

$$y = 3x^2 + x - 7$$

$$y = 2x + 3$$

$$35 = 3x + 3$$

$$\begin{aligned} 7 &= -y + x - 3x^2 \\ -3 &= -y + 2x \end{aligned}$$

$$\begin{aligned} y &= x - 7 - 3x^2 \\ y &= 2x + 3 \end{aligned}$$

$$4^2 = 3x - 3x^2$$

$$2y =$$

$$16 = x$$

$$x = 16$$

$$\begin{aligned} 7 &= 16 + 16 \\ +16 & \\ -3 &= 16 + 2x \end{aligned}$$

✗

$$y = 32 + 3$$

$$\begin{aligned} 13 &= 2x \\ x &= 6.5 \end{aligned}$$

$$-3 = 32 + -y$$

$$+35 = -y$$

$$y = 35 \quad x = 16$$

(4 marks)

9.

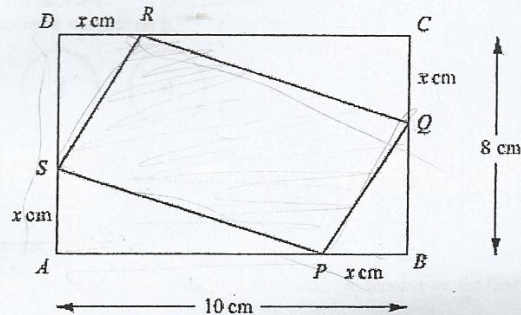


Diagram NOT accurately drawn

$ABCD$ is a rectangle.
 $AB = 10$ cm.
 $BC = 8$ cm.
 P, Q, R and S are points on the sides of the rectangle.
 $BP = CQ = DR = AS = x$ cm.

(a) Show that the area, A cm², of the quadrilateral $PQRS$ is given by the formula

$$A = 2x^2 - 18x + 80$$

$$A = 2x^2 - 18x + 80$$

~~$$A = 2x^2 - 18x + 80$$~~

$$A = 2x^2 - 18x + 80$$

~~$$A - 80 = 2x^2 - 18x$$~~

(3 marks)

10. $x = a \times 10^n$ where n is an integer and $\sqrt{10} \leq a < 10$

Find, in standard form, an expression for x^2 .
 Give your expression as simply as possible.

$$\sqrt{10} \leq a < 10$$

$$\sqrt{10} \leq a^2 < 10^2$$

$$10 \leq 81 < 100$$

$$10 \leq \sqrt{81} < 10$$

$$1 \leq \sqrt{81} < 10$$

(Total for Question 10: 3 marks)

11.

(a) Simplify

$$\frac{8(x-3)^2}{4(x-3)}$$

$$\frac{8(x^2-9)}{4(x-3)}$$
$$\frac{8x^2-3}{4x-12}$$

$$\frac{8x^2-3}{4x-12}$$

(2)

(b) Factorise

$$\sqrt{a^2-144}$$

$$\sqrt{a^2} - \sqrt{144}$$

$$a - 12$$

$$(\sqrt{a-12})(\quad)$$

(2)

(c) Make q the subject of the formula

$$p = \sqrt{q-5r}$$

$$p^2 = q^2 - 5r$$
$$q^2 = p^2 + 5r$$

$$q = \sqrt{p^2 + 5r}$$

$$q = \sqrt{p^2 + 5r}$$

(2)

(d) Solve

$$\frac{4}{y-4} = 5 + \frac{4}{y}$$

$$\frac{4}{-4} =$$

$$\frac{4}{y} = 9 + y$$

$$4 = 9y + y^2$$
$$-9y = -4$$
$$y = 2.5$$

$$y = 2.5$$

(2)

1

12. Here are seven counters.
Each counter has a number on it.



Ali puts the seven counters in a bag.
He takes, at random, a counter from the bag and does not replace the counter.
He then takes, at random, a second counter from the bag.

Calculate the probability that

- (i) the number on the second counter is 2 more than the number on the first counter,

$$\frac{2}{7} \times \frac{1}{6} + \frac{1}{7} \times \frac{2}{6} + \frac{2}{7} \times \frac{2}{6} + \frac{1}{7} \times \frac{2}{6} + \frac{2}{7} \times \frac{1}{6} + \frac{1}{7} \times \frac{2}{6} = \frac{1}{21} \quad (2)$$

- (ii) the number on the second counter is 1 more than the number on the first counter.

$$\frac{1}{7} \times \frac{1}{6} + \frac{4}{7} \times \frac{3}{6} + \frac{1}{42} + \frac{6}{42} = \frac{6}{42} \quad (2)$$

13. (a) Evaluate $5 \times 10^{12} + 9 \times 10^{12}$
Give your answer in standard form.

$$5 \times 10^{12} + 9 \times 10^{12} = 14 \times 20^{12} \quad (2)$$

- (b) Each of the numbers p , q and r is greater than 1 and less than 10

$$p \times 10^{15} + q \times 10^{15} = r \times 10^n$$

- (i) Find the value of n .

$$p \times 10^{15} + q \times 10^{15} = r \times 10^n$$

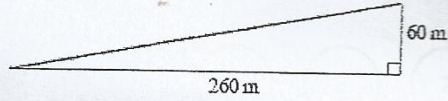
$3 \times 10^{15} + 8 \times 10^{15} = 4 \times 10^n$
 $5 \times 20^{15} = 4 \times 10^n$
 $10^n = 2^{15}$

- (ii) Find an expression for r in terms of p and q .

$$r = \dots \quad (3)$$

14.

A straight road rises 60 m in a horizontal distance of 260 m.



$$\frac{26 \times 26}{26} = 26$$

Diagram NOT accurately drawn

- (a) Work out the gradient of the road.
Give your answer as a fraction in its lowest terms.

$$\frac{60}{260} = \frac{3}{13}$$

$$\frac{3}{13}$$

- (b) Calculate how far the road rises in a horizontal distance of 195 m.

$$\frac{3}{13} \times \frac{195}{1}$$

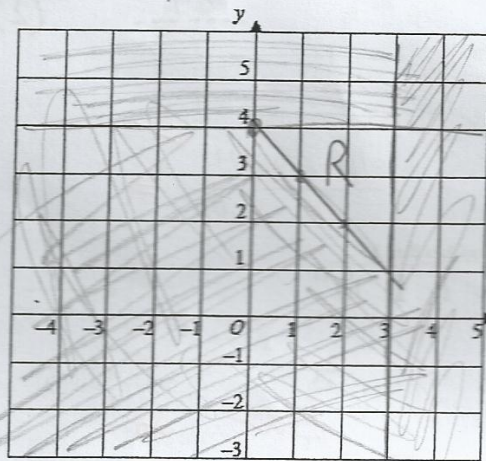
$$\frac{3.45}{13}$$

$$13 \overline{) 30.3}$$

$$\frac{30.3}{13} \text{ m}$$

(Total 4 marks)

15.



$$x + y \geq 4$$

$$\begin{array}{r} x \ 0 \ 1 \ 2 \\ y \ 4 \ 3 \ 2 \end{array}$$

(4)

- (a) On the grid, draw the line $x + y = 4$.
(b) On the grid, show clearly the region defined by the inequalities

$$x + y \geq 4$$

$$x \leq 3$$

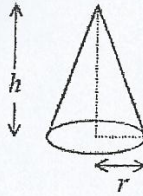
$$y < 4$$

(1)

(4)

(Total 5 marks)

16. A cone has base radius r cm and vertical height h cm.



The volume of the cone is 12π cm³.
Find an expression for r in terms of h .

$$\frac{1}{3} \pi r^2 h = 12\pi$$

$$\pi r^2 h = 36\pi$$

$$\pi r^2 h = 36$$

$$r = \sqrt{\frac{36}{h}}$$

$$r = \sqrt{\frac{36 \times h}{h^2}}$$

(Total 3 marks)

17. A box contains 7 good apples and 3 bad apples.

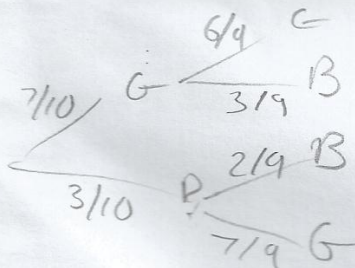
Nick takes two apples at random from the box, without replacement.

- (a) (i) Calculate the probability that both of Nick's apples are bad.

$$\frac{3}{10} \times \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$$

$$\frac{1}{15}$$

- (ii) Calculate the probability that at least one of Nick's apples is good.



$$1 - \frac{3}{10} \times \frac{2}{9}$$

$$1 - \frac{6}{90} = 1 - \frac{2}{30} = 1 - \frac{1}{15} = \frac{14}{15}$$

(4)

End of Paper (77 marks)